Notes follow and parts taken from sources in Bibliography

At the end of the 19th century, physics was considered to be almost completely solved by some. As it turned out, there were several inconsistencies which eventually led to two revolutions in physics: the theory of relativity and the quantum theory.

The most popular model of the atom in early 20th century was developed by J. J. Thomson and was known as the “plum pudding” model. In it, the electrons were tiny point particles inside a continuous distribution of positive charge, like seeds in pudding. One of Thomson’s students, Ernest Rutherford, investigated the structure of the atom in a series of experiments in 1911. Rutherford fired $\alpha$ particles (helium nuclei, emitted in some radioactive decays) at a very thin sheet of gold foil to measure the deflection angle as they went through. Because the sheet was so thin, each $\alpha$ particle could only encounter a relatively small number of atoms, and because each $\alpha$ particle was about 10,000 times as massive as an electron (the only solid thing to scatter from, as far as anyone knew), the $\alpha$ particles were expected to pass through almost completely undeflected.

Instead of this result, Rutherford found that some of the $\alpha$ particles underwent huge deflections, of sizes that were not consistent with collisions with electrons. Rutherford postulated that the positive charge holding the electrons together was concentrated into a very tiny volume (with a radius of about $10^{-15}$ m) at the center of the atom, rather than being spread out all over it.

This presented serious problems for a number of reasons. First, the Coulomb repulsion of the positive charges confined to the tiny volume of the nucleus would be enormous. At the time, there was no way to explain what other force could exist to hold the nucleus together. Additionally, something had to explain the previously determined size of the atom itself (around $10^{-15}$ m). The electrons would be pulled towards the center, leading to collapse of the atom. If the electrons were envisioned to move around the nucleus like the planets around the Sun, with angular momentum preventing the collapse, there was a different problem. A particle can only move in a circle under the influence of a force which will cause a centripetal acceleration. The problem in the case of the electron is that an accelerated charge (like one moving in a circle) must radiate energy. Calculations indicated that electrons moving in circular orbits around the large positive nucleus would radiate all their excess energy away and collapse into the nucleus in much less than one second, which obviously did not happen.

Any theory of the atom would also need to explain the fact that light emitted from isolated atoms, like those in a relatively thin gas, is only emitted at certain discrete wavelengths rather than across a continuum. The Bohr model of the atom was able to explain large parts of these problems.

In the Bohr model, the electrons would move in circular orbits without emitting radiation and spiraling in to the nucleus. The allowed orbits were restricted to those which had
discrete values of angular momentum, specifically an angular momentum equal to \( n \hbar \) where \( n \) is an integer greater than zero and \( \hbar \) is Planck's constant divided by \( 2 \pi \).

If the centripetal force holding the electron in the atom is to be supplied by the Coulomb attraction between the electron and the nucleus, we would get

\[
\frac{m_e v^2}{r} = \frac{kZ e^2}{r^2}
\]

The \( v \) is the electron's velocity in its circular orbit, and \( Z \) represents the total number of charges (equal in magnitude to that on the electron). We can solve this for \( r \) and use the fact that

\[
L = m v r = \frac{n \hbar}{2\pi}
\]

Combining these ideas, we can write

\[
r = \frac{n^2 \hbar^2}{4\pi^2 m k Z e^2}
\]

and then, since the energy of a circular orbit is well-known to be

\[
E = \frac{1}{2} m v^2 - \frac{kZ e^2}{r}
\]

we can write the energy as

\[
E = - \frac{kZ e^2}{2r} = - \left( \frac{2\pi^2 m k^2 e^4}{\hbar^2} \right) \frac{Z^2}{n^2}
\]

For the hydrogen atom (with \( Z = 1 \)), we calculate the energy for an electron in level \( n \) to be

\[
E = - 13.56 \, eV \frac{1}{n^2}
\]
Again, the restriction on \( n \) as being an integer greater than 0 still applies. When an electron moves from a higher level \( n_2 \) to a lower one \( n_1 \), it will emit one (or sometimes more than one) photon(s) with a total energy equal to \( E_2 - E_1 \).

**The Uncertainty Principle**

Another key difference between quantum mechanics and classical mechanics is the idea of **determinism**. Determinism basically says that if you knew the positions and momenta of all objects in the universe exactly, even for just an instant, you could then calculate their positions and momenta at any later time. The outcome of every coin flip, lottery drawing, football game or stock purchase after that would be known in advance. This was largely a philosophical argument, since finding the exact position and velocity of even one particle had not been done when it was proposed. Still, there was nothing to prevent this in principle – it would just take better and better measuring instruments which could be expected to be developed through technological progress.

In fact, it was later discovered that this could not be done, even for one particle, and even in principle. Heisenberg’s Uncertainty Principle says that you (or anyone else) cannot know the exact position and momentum of any one particle at the same time. It further restricts the product of the uncertainty in momentum and the uncertainty in position as shown below:

\[
\Delta p \Delta x \geq \frac{\hbar}{2}
\]

Therefore, the better you know a particle’s position (the lower \( \Delta x \)), the less you know about its momentum. You can think of this as follows: if we want to measure the position of an electron, how do we do that? At the most fundamental level, we look at it, which means we have to bounce photons off of it. In doing that, though, we transfer some momentum to the electron. If we want to measure the electron’s position to within one nanometer, for example, we need to use light with a wavelength of less than one nanometer. What momentum will the light have? We can use our previous results to see that

\[\text{if } \lambda < 10^{-9} \text{ m, } \quad p = \frac{\hbar}{\lambda} \quad \text{so} \quad p > \frac{\hbar}{10^{-9} \text{ m}}\]

Getting a better measurement of position means using light with a smaller wavelength, but that means each photon has a high momentum. Some (unknown) fraction of that momentum will be transferred to the electron, so measuring positional information causes us to alter, and therefore lose, momentum information.
This is not a problem in our macroscopic world, but it can be one on the microscopic scale. If an electron moving at 1000 m/s has a momentum of about $10^{-27}$ kg m/s, an uncertainty of $10^{-24}$ kg m/s is ridiculously large.

Of course, an uncertainty in momentum will evolve into an uncertainty in position – if the uncertainty in a particle’s velocity ($= \Delta p/m$) is 10 m/s, its uncertainty in position is growing by 10 m each second. If we localize a particle very well (measure it precisely so that its $\Delta x$ is small), the uncertain amount of momentum transferred to it may completely remove it from our viewing area. In this case, about all we can say is that a particle was at this position earlier.

There is a similar uncertainty relation for energy. An accurate determination of the energy of a state depends on how long it is observed. Basically, we are allowed to violate conservation of energy (!) very briefly and “borrow” some energy from the vacuum, as long as we return it very quickly. The product of the energy uncertainty and the time uncertainty satisfies the same inequality as above:

$$\Delta E \Delta t \geq \frac{\hbar}{4\pi}$$

It’s strange and probably a little disturbing to see that one of the things so important in physics (conservation of energy) seems to be disappearing. Before you get too worried, do a sample calculation: if you only want to borrow some energy for a picosecond ($10^{-12}$ seconds), how much can you have? Plugging $10^{-12}$ seconds in for $\Delta t$ in the formula above gives $5.3 \times 10^{-23}$ J, or about $3.3 \times 10^{-4}$ eV. Not exactly a huge amount of energy!

One of the consequences of the uncertainty principle is that certain features of classical mechanics are radically altered. For example, if we imagine a ball bouncing inside a drinking glass, it is very straightforward to find out whether it will escape or not. If the kinetic + potential energy of the ball is less than the potential energy the ball would have at the rim of the glass, it absolutely won’t get out. The uncertainty principle allows the ball to “borrow” a small amount of energy from the empty space around it and return that energy once the ball has gone over the side. Since the time that energy can be borrowed is inversely proportional to the amount borrowed, and Planck’s constant is so small, we won’t see this happen with a ball & drinking glass.

The probability of penetrating a barrier drops very quickly as the barrier’s thickness increases. Also, if the barrier is very large compared to the particle’s energy, the probability of penetration drops rapidly. In the smoke detector example, the process can be modeled by assuming the $\alpha$ particle is trapped inside the nucleus like a particle in a well. The $\alpha$ particle bounces back and forth within the nucleus until (by random chance) it grabs a small amount of energy from the vacuum, climbs the side of the well, and appears outside the confines of the nucleus. Once it’s on the outside, the repulsive force between the two protons of the $\alpha$ particle and 93 remaining ones in the nucleus cause the $\alpha$ particle to rocket away and be detected by circuitry in the smoke detector.
For this reason, the energy of the escaping $\alpha$ tends to increase as the $Z$ of the nucleus increases.

Something interesting to notice is that the energy borrowed really is returned to the vacuum. We can calculate the energy an alpha particle needs to have to escape its confinement by the nucleus, and we find that the emitted alpha particle has a lower energy than this. If the borrowed energy didn’t have to be returned (and therefore conservation of energy could be violated on the large scale), the emitted alphas would be ejected much more quickly than they really are.

This principle also explains the reason that atoms are stable. The electron can’t radiate photons and fall into the nucleus because that would make its position more certain (since the nucleus is smaller, we’d know where it is to within $10^{-14}$ m or so instead of $10^{-10}$ m), which would mean its momentum would be more uncertain, making it potentially larger. Increased momentum means increased kinetic energy. A smaller value of position, on average, would give a more negative potential energy but a more positive kinetic energy. The atom’s size is the result of a balancing act between confining an electron to a tiny volume (which the Coulomb force would like to do) and keeping its momentum and kinetic energy from increasing wildly.

Another way to look at this uncertainty involves what you do when you exercise and measure your pulse. You can count heartbeats for 10 seconds and then multiply by 6 to get your pulse rate, but the possible answers are things like 60, 66, 72, etc. You can’t get a pulse rate of 65 this way. You could count for 30 seconds and multiply by two, but you’ll only get even numbers in this case. You will get a better estimate by measuring longer. You could count for a full minute, but what if your “real” pulse rate is 67.4 beats per minute? You will still have the potential for error in the determination of the pulse rate as long as you count for a finite time.

**The Schrödinger Equation**

The requirement of quantized values of the electron’s angular momentum can be arrived at by another means. If we consider an electron with momentum $p$ to be a wave with a wavelength $\lambda = h/p$ (the idea of light as a particle gained support around the same time that the electron was postulated by de Broglie to have wave-like characteristics) and propose that an integral number of wavelengths must fit around the electron’s orbit ($2\pi r$ in size), we would get

$$\frac{nh}{p} = 2\pi r \Rightarrow mvr = \frac{nh}{2\pi}$$

which is the same condition on $r$ that was seen before. If the electron can be considered to be a wave, there must be a wave equation describing how the electron moves in general. This equation is known as the Schrödinger equation. The wave, which is
expected to be a function of position and time, is therefore known as the **wave function** and is usually represented as $\psi(x,t)$.

As a trial wave function, we can draw on what we will see will be many parallels between quantum theory and optics and try a plane wave of the form

$$\psi(x, t) = \psi_0 e^{i(kx - \omega t)}$$

The equation itself is written (in one dimension) as

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x,t)}{dx^2} + V(x,t)\psi(x,t) = i\hbar \frac{d\psi(x,t)}{dt}$$

Substituting in our plane wave solution and dividing everywhere by $\psi$, we get

$$\frac{\hbar^2 k^2}{2m} + V = \hbar \omega$$

Notice that the first term could also be written as $p^2/(2m)$, which is the classical expression for the kinetic energy of a particle of mass $m$. If we add the kinetic term to the potential term ($V$), we get the total energy, which is $\hbar \omega$, just as it is for a photon.

The Schrödinger equation just expresses $KE + PE = E_{total}$. It’s worth noticing that, since $p^2/2m$ is the nonrelativistic expression for energy, the Schrödinger equation itself is nonrelativistic. At energies where an electron’s KE is not insignificant compared to its rest mass energy, a relativistic equation is required (known as the **Dirac equation** for the electron).

Because the Schrödinger equation contains the first derivative with respect to time, but the second derivative with respect to position, it looks very similar to a **diffusion equation**, which specifies how two materials mix (a drop of milk in a glass of water, for example). The weird thing about the Schrödinger equation is that it is a diffusion equation in **imaginary** time (notice the “$i$” on the right side). What is diffusing in this case? The wave function itself, which describes the electron’s position/momentum.

Because the absolute square of the wave function is proportional to the chance of finding the electron in any given region, we can say that this expresses what we saw earlier with the uncertainty principle: after we locate the electron, its position gradually gets less certain with time (due to $\Delta p$) as the wave function spreads out, or diffuses, through space.
Solving the Schrödinger Equation

The Schrödinger equation is usually given in either time dependent or time independent form. The example above is the time dependent case – the potential is a function of position as well as time, and the wave function is also. The time derivative on the right is the signature of time dependence.

In the cases we will examine, the time independent Schrödinger equation will be good enough. This means that the potential will no longer be a function of time, and that the time derivative term on the right side of the equation will be replaced by $E$, the energy of the particle. The energy is a constant in this case, because time independence is deeply connected to the conservation of energy. In the same way, translational invariance is deeply connected to the conservation of momentum (if you're interested in learning more about this, it's called Noether's theorem). We now get the simpler form

$$\frac{-\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

known as the Time Independent Schrödinger Equation (TISE).

If we want to begin to use this to solve problems in quantum mechanics, we can start with the problem of a one-electron atom. First, we need to rewrite the 1-D equation above in three dimensions. We also need to come up with an expression for the potential felt by the electron. Finally, just as you may have done in your first semester physics course if you proved Kepler's third law, we have to account for the fact that, in a classical situation, the electron and the nucleus both move. The electron will move much more than the nucleus, but to make the math easier, we can define a reduced mass which we will use instead of the electron's mass. By effectively changing coordinates to a system where the electron does all of the moving, we don't need a term for the kinetic energy of the nucleus.

If a mass $m_1$ and a mass $m_2$ are in orbit around one another, we can define the reduced mass as

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

We'll use this $\mu$ in place of the $m$ in the time independent Schrödinger equation.

The potential is just what you would have written in the first weeks of the second semester of physics for the potential energy between two charges:
\[ V(x, y, z) = -\frac{Zk e^2}{\sqrt{x^2 + y^2 + z^2}} \]

where \( Z \) is the number of protons in the nucleus, \( k \) is the constant from Coulomb’s law, \( e \) is the magnitude of the charge on an electron/proton, and \( r \) is the atomic radius.

Finally, we have to rewrite the kinetic term in the Schrödinger equation in three dimensions:

\[ -\frac{\hbar^2}{2m} \nabla^2 \psi(x, y, z) \]

While it would be simple to write this new equation in Cartesian coordinates, that would ignore the natural symmetry of the problem. The Coulomb potential has spherical symmetry, so solving it will be much easier if we switch to spherical coordinates.

Writing out the full equation (including expanding the Laplacian operator in the kinetic term) gives us

\[ -\frac{\hbar^2}{2\mu} \left( \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) \psi + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right) - \frac{Ze^2}{r} \psi = E\psi \]

where \( \psi \) is a function of all three variables.

The usual method for solving this equation (and the reason the spherical coordinate system is preferred) is that the wave function is separable into a product of functions, each of which depends only on one coordinate. In other words, we can restrict ourselves to solutions that satisfy \( \psi(r, \theta, \phi) = R(r) \Theta(\theta) \Phi(\phi) \).

When we substitute this compound form for \( \psi \) into the TISE, we can look for solutions to the individual pieces of the wave function, which is a much easier problem. We can rewrite the equation above as

\[ \frac{1}{R(r)\Theta(\theta)\Phi(\phi)} \left( \sin^2 \theta \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) \Phi + \sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Phi}{\partial \theta} \right) + \frac{\partial^2 \Phi}{\partial \phi^2} \right) = -\frac{2\mu}{\hbar^2} \left( E + \frac{Ze^2}{r} \right) \]

and look at the behavior of the compound form of \( \psi \) in the derivative with respect to \( \phi \), for example:
\[
\frac{\partial^2 \psi (r, \theta, \phi)}{\partial \phi^2} = R(r) \Theta (\theta) \frac{d^2 \Phi (\phi)}{d \phi^2}
\]

where we have done away with the partial derivative. We can move all terms containing \( \Phi(\phi) \) or its derivatives to one side of the TISE, and all other terms to the other side. This means one side contains all of the \( \phi \) dependence and the other contains all of the \( r \) and \( \theta \) dependence. That means (and this is a frequently used technique in physics) that each side must be equal to a constant (which we call \( -m^2 \) by convention). The \( \Phi(\phi) \) containing side can then be written as

\[
\frac{1}{\Phi (\phi)} \frac{d^2 \Phi (\phi)}{d \phi^2} = -m_i^2 \implies \frac{d^2 \Phi (\phi)}{d \phi^2} = -m_i^2 \Phi (\phi)
\]

The solution to the equation above is clearly an oscillating exponential of the form

\[
\Phi (\phi) = e^{im_i \phi}
\]

The forms of the solutions for \( R(r) \) and \( \Theta(\theta) \) are unfortunately not quite so simple. The solution for \( \Theta(\theta) \) can be written in terms of special mathematical functions known as Legendré polynomials, which are polynomials in \( \cos(\theta) \) such as \( \cos(\theta) \) itself (the first Legendre polynomial), \( \frac{1}{2} (\cos^2 \theta - 1) \) (the second), etc. The \( n^{th} \) Legendre polynomial will have a term proportional to \( \cos^n \theta \) in it.

The \( R(r) \) solutions are even weirder things known as hypergeometric functions. Rather than work through this, we will look them up in tables.

**Hydrogenic Wave Functions**

There are three numbers that will characterize the solutions described above – one for the \( \Phi(\phi) \) part (the \( m_i \) mentioned earlier, known as the magnetic quantum number, which is also part of the solution for \( \Theta(\theta) \)), one that is used for \( \Theta(\theta) \) and \( R(r) \) (\( l \), the angular momentum quantum number), and one used only for \( R(r) \) (\( n \), the principal quantum number).

The mathematical properties of the functions above place restrictions on the values of \( n \), \( l \), and \( m_i \) as follows

\[
\begin{align*}
n &= 1, 2, 3, \ldots \\
l &= 0, 1, 2, \ldots n-1 \\
m_i &= -l, -l+1, \ldots l-1, l
\end{align*}
\]
The ground state of the Hydrogen atom therefore has \( n = 1 \), which forces it to have \( l = m_l = 0 \). It spends most of its time near the nucleus (i.e., the center of the atom).

A few wave functions for various values of \( n, l, m_l \) are shown below (from the Eisberg & Resnick book)

<table>
<thead>
<tr>
<th>( \psi ) ( n l m_l )</th>
<th>Wave function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi 100 )</td>
<td>( \frac{1}{\sqrt{\pi}} \left( \frac{Z}{a_0} \right)^{3/2} e^{-Zr/a_0} )</td>
</tr>
<tr>
<td>( \psi 200 )</td>
<td>( \frac{1}{4\sqrt{2\pi}} \left( \frac{Z}{a_0} \right)^{3/2} \left( 2 - \frac{Zr}{a_0} \right) e^{-Zr/2a_0} )</td>
</tr>
<tr>
<td>( \psi 210 )</td>
<td>( \frac{1}{4\sqrt{2\pi}} \left( \frac{Z}{a_0} \right)^{3/2} \left( \frac{Zr}{a_0} \right) e^{-Zr/2a_0} \cos \theta )</td>
</tr>
<tr>
<td>( \psi 21-1 )</td>
<td>( \frac{1}{8\sqrt{\pi}} \left( \frac{Z}{a_0} \right)^{3/2} \left( \frac{Zr}{a_0} \right) e^{-Zr/2a_0} \sin \theta \ e^{-i\phi} )</td>
</tr>
</tbody>
</table>

The \( a_0 \) represents the **Bohr radius**, which is about \( 5.29 \times 10^{-11} \) m. Notice that the wave function for the \( 100 \) state drops off exponentially as you move away from the nucleus, meaning the chance of finding it is largest at \( r = 0 \).

In the \( 210 \) state (the first one with angular momentum), the spherical symmetry that was present in the \( 100 \) and \( 200 \) wave functions (which had no \( \theta \) or \( \phi \) dependence) is now gone. The \( 210 \) state has a \( \cos \theta \) dependence in addition to the radial dependence. The \( 21-1 \) state features \( \phi \) dependence as well.

Because the wave function represents a probability **amplitude**, though, we have to multiply it by its complex conjugate (i.e., the same wave function but with all \( i \) terms replaced by \(-i\). That will remove the \( \phi \) dependence. Because the probability of finding the electron at any infinitesimal point will be zero, we can only talk about probability densities. We need to integrate \( \psi^* \psi \) over a finite volume to get a probability. As you can see, the dimensions of the wave function are meters\(^{-3/2}\), so the square of the wave function integrated over a volume will give a dimensionless quantity, which probability must be.

Because the Coulomb potential itself is spherically symmetric, there can be no energy dependence on either \( l \) or \( m_l \). Only the presence of a magnetic or electric field breaks this symmetry.
The restrictions on \( l \) and \( m_l \) explain the arrangement of the periodic table of the elements. No two electrons can be in the same state (i.e., no two can have the same quantum numbers). To completely describe the electron’s state requires the introduction of two more quantum numbers, however. The value of the spin angular momentum quantum number \( s \) is always \( \frac{1}{2} \) for an electron (for this reason, it is also known as the electron’s intrinsic angular momentum, or the angular momentum it has just because it is an electron). The other quantum number is \( m_s \), and it has the same relationship to \( s \) that \( m_l \) has to \( l \): it ranges from \(-s\) to \(+s\) in integer steps. Of course, that means there are only two choices for \( m_s \), and they are \( \pm \frac{1}{2} \).

This is the reason the first row of the periodic table contains only two elements – H and He. The ground state of the H electron is the \( 1s^1 \) state. The He electrons are both in the \( 1s^2 \) state, but this is allowed because each electron has a different value of \( m_s \). In the second row, lithium has its third electron in the \( n = 2 \) state. There are a total of 8 elements in the second row, with neon having two electrons (one of each spin) in the \( 2s^2 \) state, two in the \( 2p^2 \) state, two in the \( 2p^1 \) state, and two in the \( 2p^1 \) state.

This is the reason that the noble gases (which all have full outer shells) are so reluctant to combine with other elements – there are no remaining states in the full shell, and the atom itself is electrically neutral and not attractive to other electrons as a result.

**Nuclear Physics**

The protons and neutrons inside the nucleus see a very different environment than what the electrons can see. The electrons moving in the atom are dominated by the effects of the electromagnetic force. Inside the nucleus, the electromagnetic repulsion of the protons is very large. Using Coulomb’s law and a separation between protons of 1 Fermi, we can calculate a repulsive force of over 100 N – an incredibly large force acting on such a tiny mass. Also, we can calculate an electrostatic potential energy of over 1 MeV – far larger than the few eV binding a typical outer-shell electron to an atom.

Clearly, there is a much stronger force binding the protons and neutrons to one another. This force is also only important over very short distances, having a typical range of approximately 1 F. If we assume that forces are mediated by the exchange of particles (just as the electromagnetic force is mediated by the exchange of photons), we can use the uncertainty principle to estimate the mass of the messenger particle. A particle with a rest mass of \( m \) will have a minimum energy of \( mc^2 \), meaning that if it appears from the vacuum via the uncertainty principle, it must return to the vacuum within a time

\[
\Delta t \leq \frac{\hbar}{2mc^2}
\]

With this limit on its lifetime, we can then say that it cannot move faster than the speed of light, so its range is limited by \( x = c \Delta t \). Following the same process backwards and
using 1 fm as our maximum range, we predict a rest mass energy of approximately 100 MeV.

Using this line of reasoning, experimenters searched for and found the pion (actually, three different pions – the $\pi^+$, $\pi^-$, and $\pi^0$). A proton emitting a $\pi^+$ will become a neutron, as a proton absorbing a $\pi$ will also. The $\pi^+$ and $\pi^-$ are antiparticles of each other, while the $\pi^0$ is its own antiparticle. The actual rest mass of the $\pi^+$ and $\pi^-$ is about 140 MeV/c$^2$, and about 135 MeV/c$^2$ for the $\pi^0$. Within the nucleus, we can imagine a flood of these pions throughout, stabilizing the collection of protons and neutrons.

The pions are scalar particles, meaning they have spin 0. Massive scalar particles have their own wave equation, known as the Klein-Gordon equation. This is a relativistic equation, but we can look at the time-independent case and if we solve it, we get the Yukawa potential:

$$V = \frac{\alpha e^{-r/\lambda}}{r}$$

where the $\lambda$ represents the typical range of the potential and $\alpha$ is the coupling constant that describes the strength of the interaction. Notice that we could apply this idea to the photon: if its rest mass is zero, the effective range of the interaction is infinite. The reason is that, for any particle separation, there is always a photon with a low enough energy to make the trip between the two particles. The range limit for the exchange of a massive particle arises because, in that case, there is a minimum energy – the energy required to produce the particle.

In the photon case, if the range is infinite, the exponential term in the potential above will approach one. The potential then becomes $\alpha/r$, which is in fact just the ordinary Coulomb potential.

**Binding Energy and Mass Defect**

Because the protons and neutrons are bound to each other in the nucleus, they are in a lower energy state than if they were to be separated from each other. This means that energy would have to be added to the nucleus to split it into individual particles, and since energy and mass are connected via Einstein’s formula $E^2 = p^2c^2 + m^2c^4$, a nucleus of $Z$ protons and $N$ neutrons actually has a mass less than $Zm_p + Nm_n$. The difference between the actual nuclear mass and the mass of the parts that make it up is known as the mass defect.

The mass defect can be multiplied by $c^2$ to give the total binding energy of the nucleus. If we then divide that by the number of nucleons ($Z+N$ in the previous paragraph), we are left with the binding energy per nucleon. This is a measure of the stability of a given nucleus. We can make a curve of the binding energy per nucleon as a function of
nucleon number, and it looks like the graph below (from [http://www.lbl.gov/abc/wallchart/chapters/02/3.html](http://www.lbl.gov/abc/wallchart/chapters/02/3.html)):

![Graph](https://via.placeholder.com/150)

Two of the notable features of the curve are 1) the peak at $A = 4$ (representing $^4$He, having two protons and two neutrons) and 2) the more gradual (and highest) peak around $A \sim 60$. The presence of the higher peak is very important for the process of creation of the elements in the cores of stars, known as **stellar nucleosynthesis**.

At the center of stars such as the Sun, gravitational forces contracting the star increase the temperature within it. Higher temperatures mean higher energies for the particles in the region. Because the Sun (and the universe) is mostly hydrogen, and the high temperatures at the core of a star are far more than necessary to ionize hydrogen, the interior of a star is a plasma. The protons moving around at high speed can collide with enough energy to overcome the repulsive Coulomb force. The distance of closest approach depends on the kinetic energy of the protons, which in turn depends on the temperature.

When the protons get close enough for the pion exchange force to be felt, they will undergo **nuclear fusion**, combining in the process below:

$$p + p \Rightarrow ^2\text{H} + e^+ + \nu_e$$

The emission of the positron is to conserve charge as one of the protons becomes a neutron, and the neutrino is emitted because **lepton number** also must be conserved. The known leptons are the charged electron, muon, tau particle, the three neutral neutrinos (identified with each charged particle) and the six antiparticles of these. The particles have lepton numbers of +1 while the antiparticles have lepton numbers of -1.

This is just the first step in the **proton-proton chain** that turns 4 protons into the helium-4 nucleus and releases energy. The energy is released because the $^4$He nucleus
has a large binding energy, meaning it must get rid of a large amount of energy to exist. (In larger stars, a similar process turns hydrogen into helium, but does so using a carbon nucleus as a catalyst. This is known as the CNO cycle.)

The energy released by this fusion is about 27 MeV. Notice that this is approximately ten million times more energy than we would expect to be released in a chemical reaction, which essentially shuffles electrons around, because electronic binding energies are typically a few eV. This is the reason that hydrogen bombs (well under a ton of active material) typically have energy yields equivalent to millions of tons of the chemical explosive TNT.

In a star, the heat and pressure produced by fusion keeps gravity from crushing the star. When the hydrogen is exhausted, the star will contract and temperatures will rise (just as you notice when pumping up a bicycle tire – compressing a gas makes it hotter). Eventually (if the star is not too small), the temperature will rise to the point that helium nuclei can fuse with one another. There is a particularly energetically favorable method for combining them, where three $^4\text{He}$ nuclei form a single nucleus of carbon-12, known as the triple-alpha process.

Once the helium is scarce, the cooling and gravitational compression resume. In large enough stars, the temperature can increase until $^{12}\text{C}$ starts to fuse with other nuclei (remaining $^4\text{He}$ that didn’t find other nuclei to fuse to). Of course, the temperature has to go up for this to happen because the repulsive force between a $^{12}\text{C}$ and a $^4\text{He}$ is much larger than that between two $^4\text{He}$ nuclei.

This process continues in large stars, forming larger and larger nuclei. Since helium is so abundant, it is common for the elements to grow by 2 protons and 2 neutrons at a time. Since atomic number is equal to the number of protons, we can look at a chart of elemental abundances in the solar system and notice an interesting trend. Of course, there is a decrease in abundance as atomic number increases since the heavier elements are harder to make. We might then expect that the abundance of an element with $Z$ protons will always be smaller than one with $Z-1$ protons. This is true for helium and hydrogen, certainly. However, take a look at the plot below:
Most of this graph is above the line where the log of the abundance ratios is zero. This means that elements with even numbers of protons are more abundant than those with odd numbers, and the reason is that most elements are built up two protons (and two neutrons) at a time. Also, notice that the even-Z elemental abundance is compared to the abundance of the lower odd-Z neighbor. The general trend is that abundance decreases with Z, so this is stronger evidence of the addition of helium nuclei as basic building blocks.

The buildup of elements can’t continue forever. As the nucleus grows, we move past the peak of the binding energy curve at around A ~ 60. In a star, $^{56}$Fe is the energetic endpoint. Further fusion between helium nuclei and heavier nuclei can still happen, but since it will move the nucleus down the binding energy curve, it is energetically unfavorable. Fusion by the addition of single neutrons to a nucleus is still possible – the neutrons do not feel any electromagnetic repulsive force, so they are able to get very near the nucleus, until the strong force grabs them and traps them. When a nucleus has too many neutrons, it will start to convert some of them into protons through the process of beta decay, which we will examine soon. Heavy elements up to about $^{208}$Bi can be formed through this slow method (known as the s-process).

The heavier elements are believed to form when a star’s internal furnace finally runs out of useful fuel (i.e., the core is mostly iron) and the heat and pressure produced by fusion start to decline. Gravity keeps working, however, and the star’s core begins to collapse inward. When the electrons and protons remaining in the core are compacted under high enough gravitational pressure, they form neutrons. The neutron degeneracy pressure is much larger than the electron degeneracy pressure which holds up white dwarf stars. The outer core of the collapsing star will be racing inward at a reasonable fraction of the speed of light during the collapse, and when it hits the hard boundary of the neutron core, it rebounds.
This rebound creates a flood of neutrons which will irradiate the star’s material very rapidly (known as the r-process) and produce elements up to at least $^{238}\text{U}$. The subsequent explosion scatters the heavy elements throughout space, to be incorporated in later generations of planets and stars.

The fusion process, if controlled on Earth, would provide an almost unlimited source of energy since we would be fusing hydrogen, freely available by splitting water. The problem is that, while a star has tremendous gravity to hold the high-speed, high-temperature nuclei in place, we don’t. We need something that will confine the fusing nuclei at temperatures over one million Kelvin. No physical container can do this without instantly vaporizing and, at the same time, cooling off the reaction. One of the current ideas is to try to confine the fuel using magnetic fields, known as magnetic bottles.

**Nuclear Fission**

At the far end of the binding curve, we find the heaviest elements. While it is energetically unfavorable to try to get huge collections of protons (which repel each other very strongly) close enough to fuse, it turns out that breaking large nuclei into smaller (more tightly bound) ones will also release energy. We can cause this breakup to occur in many materials by firing a neutron at the nucleus. The problem with doing this randomly is that a nucleus a) might absorb the neutron and keep it, making an isotope of the target element (that is, a nucleus with the same number of protons but a different number of neutrons) b) the nucleus might split into two roughly-equal pieces (fission), but that would be the end of the reaction since those pieces, which will have a large positive charge, are unlikely to be able to break up any other nuclei or c) the nucleus might split and release more neutrons. This is the most desirable outcome, because the neutrons released have a good chance of causing more fissions, which will release more neutrons, etc. This is known as a chain reaction, where the number of fissions grows exponentially.

One naturally occurring candidate for fission is $^{235}\text{U}$. The problem with obtaining it is that most uranium is $^{238}\text{U}$, which will not keep a chain reaction going. During WWII, huge amounts of money and effort were spent to separate the two isotopes. Because they are both uranium, they can’t be separated chemically, as could be done for two different elements in a compound. The separation had to be physical, based on the mass difference. Of course, the mass difference was about 3/238 \sim 1\%, so this was very difficult as well. Adding to the difficulty, the uranium had to be chemically attached to fluorine in the form of UF$_6$ to form a gas. The added mass of 6 fluorine atoms did not make separation easier.

The two most common methods at the time involved centrifuges, where the uranium hexafluoride could be spun rapidly until the slightly heavier $^{239}\text{U}$ concentrated at the outside of the centrifuge. In a similarly involved process, the same gas was sent through a series of membranes in a diffusion plant. The lighter $^{235}\text{UF}_6$ would get to the end of the diffusion stage a little more quickly, on average, than the heavier $^{238}\text{UF}_6$. After many stages, the uranium was relatively refined.
It was also discovered that $^{239}\text{Pu}$ was a good fission candidate, so the government also had teams working on that kind of bomb. Since plutonium has such a geologically short half-life, there are no deposits of it to mine. It had to be created by bombarding other materials with neutrons.

**Nuclear Decay**

Radioactive decay is a nuclear process rather than an atomic process. This decay occurs when a nucleus is somehow out of balance; there are too many protons, or neutrons, or both, or there is too much energy in the nucleus. The three most common types of radioactive decay particles are alpha, beta, and gamma. The $\alpha$ particle consists of two protons and two neutrons which are bound together very tightly. This can also be thought of as the nucleus of a helium atom, or He$^{++}$. The $\beta$ particle is most commonly thought of as an electron, although the positron is also considered a $\beta$ particle (it’s generally a good idea to remove all ambiguity by referring to the electron either by name or with the symbols $e^-$ or $\beta^-$ and the positron either by name or with the symbols $e^+$ or $\beta^+$). Finally, the $\gamma$ is just a high-energy photon

If you examine the periodic table, you’ll notice that elements near the top tend to have weights that are about twice their atomic numbers, meaning the number of protons and neutrons is equal. This makes a stable nucleus, at least for small nuclei. It’s worth mentioning that the atomic weight is not usually an integer for a few reasons, including mass defect as well as the fact that it is an average of $A$ weighted by abundance of the different isotopes.

For larger elements, the number of neutrons exceeds the number of protons in a stable nucleus. The reason is that the pion exchange force has a very short range, so the nucleons are effectively bound due to interactions with their immediate neighbors. Adding more nucleons will increase the binding for other nucleons that weren’t already surrounded, but it doesn’t make much difference for those that were. Because the electromagnetic force has infinite range, though, every proton in the nucleus repels every other proton, tending to make larger nuclei (with more protons) less stable. The addition of neutrons allows the pion exchange force to continue to stabilize the nucleus while keeping protons further away from one another and thereby reducing the repulsive forces.

When the atoms are so big that the electromagnetic repulsion starts to win, it is common for them to emit an alpha particle. This gets rid of two neutrons, which doesn’t help much, but it also gets rid of two protons, so it’s worth it energetically since it improves the neutron-proton ratio. This is a very common decay mode for elements at the bottom of the periodic table.
### β decay

The remaining kind of nuclear decay for us to examine is β decay. The three types of β decay are known as **electron emission**, **positron emission**, and **electron capture**. Each of these processes will change Z by one unit as a proton or neutron becomes a neutron or proton. The value of A does not change during β decay. If the balance of neutrons and protons is displaced from its ideal value, β decay can bring it back to a more stable ratio.

This process is mediated by the weak nuclear force, which we will discuss later. We can describe this decay by

$$ n \rightarrow p + e^- + \bar{\nu}_e $$

The decay above satisfies conservation of charge, lepton number, baryon number, and mass-energy. We’ll look at baryons again after learning about quarks, but for now, you can just remember that neutrons and protons both have a baryon number of +1 and their anti-particles have a baryon number of -1. Leptons have a baryon number of zero, just as baryons have a lepton number of zero. This decay is also what happens to an isolated neutron. Oddly enough, the neutron is only stable when it is part of a nucleus. If left alone, half of a sample of isolated neutrons will decay in about 10 minutes.

Nuclei can also decay via positron emission, also called β⁺ decay. This process is similar to the one above, but now we have

$$ p \rightarrow n + e^+ + \nu_e $$

Notice the changes relative to the first decay. Since the proton has a positive charge on the left, something on the right must have a positive charge, which is why we get a positron rather than an electron. Also, since the positron has a lepton number of –1, this is balanced by an electron-neutrino rather than an antineutrino.

Unlike in the case of the neutron, this process **cannot** happen outside of a nucleus, since the total mass-energy on the right is larger than that on the left and would not be conserved. This is only possible when the proton can “borrow” energy from somewhere else, like the nucleus to which it belongs. As far as anyone has been able to determine, protons are stable. If proton decay is even possible, as some unified theories predict, experiments show that the half-life of the proton must be larger than at least $10^{32}$ years!

Finally, just as β⁺ decay attempts to fix the problem of having too many protons for the number of neutrons in a nucleus, electron capture is another way of doing the same thing (also known as a **competing process**). After positron emission, the atom will
have lost an energy of at least $2 \times 511,000$ eV, or about $1$ MeV (half of this goes into the positron that is created, and the other half represents the electron that the atom will lose). If a nucleus has too many protons but energy of the product nucleus (also called the daughter) that would be produced by positron emission is less than about $1$ MeV below the parent nucleus, electron capture is the only way the proton ratio can be adjusted.

This is most common in heavy elements such as $^{83}$Rb. In these elements, the outer shell electrons tend to compact the inner electron orbitals so that the innermost electrons spend a lot of time inside the nucleus itself, making capture that much easier. This competing process is undesirable from an imaging standpoint, since the PET scanner is looking for positrons rather than evidence of electron capture (abbreviated as EC). After EC has occurred, there will of course be an inner shell vacancy, so an electron from a higher shell will quickly move to fill it.

**γ Decay**

The other common decay mode is γ decay. If two nuclei have the same number of neutrons and protons but different energies (i.e., one is in an excited state), they are called isomers and the excited state is usually represented by putting an “m” for metastable after the “A” superscript:

$$^{99m}Tc \rightarrow ^{99}Tc + \gamma$$

The neutrons and protons in a nucleus can be thought of as occupying energy levels or shells, just as the electrons around an atom do. The major difference is that, while electronic transitions may involve energies of a few eV, nuclear transitions typically involve energies of millions of eV.

If the nucleus ends up in an excited state (usually as a result of a previous α or β decay), it will emit a γ to get back to its ground state. The γ can escape completely or it can interact with one of the atomic electrons on the way out. If it hits one of the electrons, it will certainly ionize it, and this process is called internal conversion. This is analogous to the production of an Auger electron when X-rays are involved. In each case, the photon can either leave without interacting or it can give all its energy to an electron, ionizing it in the process. For imaging purposes, giving the energy to an electron is not good. Our equipment is designed to catch photons, and any competing process will only add to the patient’s radiation dose without aiding image quality.

Gamma rays are, in general, the most penetrating of the three types of radiation we have seen. Depending on the energy of the photon, the γ can penetrate large thicknesses (meters, in some cases) of concrete or steel.
Nuclear Models

The two primary models used to describe events within the nucleus are the shell model and the liquid drop model. The liquid drop model treats the nucleus as if it were a drop of water. There are competing forces in an actual drop of water – there is surface tension, which arises because molecules at the surface of the drop (analogous to nucleons at the exterior of the nucleus) only feel attractive forces from their neighbors, but feel nothing on their exposed sides.

This semi-empirical formula (found in the Eisberg and Resnick book in the bibliography) for nuclear stability has six terms and calculates the mass of the nucleus. From that, the mass defect can quickly be found, and we know that the binding energy is equal to the mass defect multiplied by c^2. The first term in this formula is therefore just the mass of the constituent nucleons.

Next, the volume term that predicts larger nuclei are more stable - it can be written as \(-a_1 A\) where \(a_1\) is a constant and \(A\) is the number of nucleons (= N + Z). The surface tension term is a correction to this overestimate of binding energy which assumes every nucleon is surrounded by neighbors. This is therefore a positive term, reducing binding energy. Since the nuclear radius is proportional to \(A^{1/3}\), the surface area is proportional to \(A^{2/3}\), so we have a term that can be written as \(+a_2 A^{2/3}\).

There is also a term representing the Coulomb repulsion of the protons. In general, the electrostatic repulsion energy could be written as \(q_1 q_2 / r\). A nucleus with \(Z\) protons will then have a repulsive energy of \(+a_3 Z^2 / A^{1/3}\). The fact that nuclei generally prefer to have \(Z = N\), at least for \(Z\) and \(N\) relatively small, is reflected in the next term which is \(+a_4 (Z - A/2)^2 / A\). The square means that this term is always positive, reducing nuclear stability as it increases. If \(Z=N\), though, it will disappear.

Finally, there is a term that represents the preference for even values of \(Z\) and \(N\). This term is zero if one of the two (\(Z\) or \(N\)) is odd and one is even. If both are even, the term is negative, increasing the binding energy. If both are odd, the term is positive, decreasing the binding energy. It usually fits the data best if it is multiplied by \(A^{-1/2}\).

This model gives reasonably good results in many cases.

The Shell Model

The shell model shares some features with the standard atomic model which is so successful at explaining the periodic table – it assumes that there are energy levels (shells) which can only hold a certain number of electrons, and an atom with a filled outer shell is more stable chemically than one with an unfilled shell.

Similarly, there are certain special numbers (known as “magic numbers”) of neutrons or protons that make a nucleus less likely to decay or increase its lifetime when compared to nuclei with non-magic numbers of neutrons or protons. A nucleus with a magic...
number of both neutrons and protons is known as **doubly magic** and is generally very stable. The first example of this is the helium nucleus, which has 2 neutrons and 2 protons. As in the electronic shell case, 2 is a magic number, and helium is so stable that it is common for an out-of-balance nucleus to eject a whole helium nucleus (alpha decay) rather than just part of one.

In general, even numbers of neutrons and even numbers of protons are more stable than odd numbers (this would help explain the earlier graph of elemental abundances). The magic numbers are 2, 8, 20, 28, 50, 82, and 126. Examples of doubly-magic isotopes would be $^4\text{He}$, $^{16}\text{O}$, $^{40}\text{Ca}$, $^{56}\text{Ni}$, etc. The shell model is based on the strong interaction between orbital and intrinsic angular momentum in the nucleus (sometimes known as $L\cdot S$ coupling).

**Quarks, etc.**

A more complete explanation of the nuclear interior was formulated in the 1960's-1970's. The nucleons were no longer believed to be fundamental particles, but were thought to be composed of smaller particles, known as **quarks**. There are three families of quarks, with each family containing a pair of quarks. These families are

$$
\begin{pmatrix}
u \\
d \\
c \\
t \\
s \\
b
\end{pmatrix}
$$

The upper quark in each family has a charge of $+2e/3$ and the lower quark has a charge of $-e/3$. Notice that the fundamental unit of electric charge, which has always been $e$, is now apparently $e/3$. As it turns out, these fractional charges are never observed. The quarks are always found either in groups of three (as in the neutron and proton) or in quark-antiquark pairs known as **mesons** (the different $\pi$ particles mentioned earlier are mesons). When three quarks are bound together, we call the resulting particle a **baryon**. Protons and neutrons are baryons, with the proton being composed of two up quarks and a down quark while the neutron is a pair of down quarks and an up quark. Conservation of baryon number is one of the things that is required in nuclear decays, such as the transformation of a neutron to a proton in $\beta$ decay.

The force that holds the quarks together as mesons or baryons is known as the **strong nuclear force**, and it is the strongest of the four fundamental forces. It is sometimes also called the **color force**. The reason is that quarks, in addition to having one of the six **flavors** above, will also have one of three **colors** (typically chosen to be red, green, and blue). Of course, these things which are far smaller than the wavelengths of visible light don’t have color in the sense we’re used to thinking of. The reason this property is called “color” is because you can consider all observable particles to be colorless, or “white”. It takes red, green, and blue together to make white, so each quark in a baryon will have a different color. In the mesons, the quark will have one color (red, for
example), and the antiquark will have the opposite color (antired), leaving a colorless meson.

Just as we thought of neutrons and protons changing identities when they exchange pions, we can think of quarks changing colors when they exchange **gluons**. These are the carriers of the color force, just as photons are the carriers of the electromagnetic force. There are quite a few varieties of gluon, since they can be things like red-antigreen, blue-antired, green-antiblue, etc.

The nuclear force that we have been talking about up until now is really a kind of “leftover” of the color force holding individual nucleons together. It’s similar to the way that van der Waals forces hold some molecules together, when they are really only the residuals of the electromagnetic force holding the individual atoms together.

The color force has a short range, but it also exhibits something called **asymptotic freedom**. This works sort of like the way a leash works for a dog: over a certain range, it exerts (almost) no force on the dog, but at the limit of the leash, the force suddenly gets very large. This is one of the problems involved in a quantum theory of the color force, known as **quantum chromodynamics**. In quantum electrodynamics, which describes how the photon mediates the force between charges, the force dies off with distance, which means a power expansion is useful since each higher-order term should be less important than the previous one. If the force effectively **increases** with distance, the higher-order terms won’t disappear.

In an effort to clear up the various names used for the particles so far, we can make the chart below:
Anti-baryons (with baryon number = -1) and anti-mesons would just replace quarks with antiquarks. The anti-proton, therefore, is made of two anti-up quarks and one anti-down quark. The antiparticles of mesons are just other mesons, since the neutral pion (π⁰) is its own antiparticle and the π⁺ and π⁻ are each other’s antiparticle.

Leptons

The leptons are (in general) the lighter matter particles. Again, there are three families of lepton, and they are shown below:

\[
\begin{pmatrix}
  e^- \\
  \nu_e
\end{pmatrix}
\begin{pmatrix}
  \mu^- \\
  \nu_\mu
\end{pmatrix}
\begin{pmatrix}
  \tau^- \\
  \nu_\tau
\end{pmatrix}
\]

Each of the upper leptons has a charge of \(-e\) and each of the lower leptons (the neutrinos) is electrically neutral. All of these have a lepton number of +1. Each family has larger masses than the family to its left. For this reason, we can think of the \(\mu\) and \(\tau\) particles as “heavy electrons”. The neutrinos, though, are very odd particles. They have only the tiniest chance of interacting with matter, and almost every neutrino produced by the Sun and aimed at the Earth will pass through the whole planet without interacting.

After many years of careful measurement, it has been determined that neutrinos have rest masses well under 1 eV (the electron, the smallest non-neutrino matter particle, has a mass of 511,000 eV). All quarks and leptons have antiparticles, and the neutrinos are no exception. Just as antibaryons have a baryon number of -1, antileptons have a lepton number of -1.

The leptons don’t feel the color force, but they do feel another nuclear force, known as the weak nuclear force. This force is responsible for beta decay, among other things. The messenger particles for this force (like the gluon for the color force and the photon for the EM force) are the \(W^+, W^-, Z^0\).

Notice that the \(W\) particles are charged, meaning they will also participate in the EM interaction. In fact, physics now considers the four fundamental forces (strong, weak, EM, and gravity) to have been partially unified into the strong, electroweak, and gravitational forces. One of the ultimate goals of physics is to continue this process of unification of the forces until all forces can be understood as related aspects of one force, just as electricity and magnetism are no longer considered to be separate forces but are now understood to be parts of the electromagnetic force.

We can now take a closer look at the \(\beta\) decay of a neutron. Instead of just

\[n \rightarrow p + e^- + \bar{\nu}_e\]
we can look at the quark level. We must have a down quark becoming an up quark to
go from the (udd) of a neutron to the (uud) of a proton. 

\[ d \rightarrow u + ? \]

To find out what the question mark above represents, consider that we have to have
conservation of charge. This means the missing particle must have a charge of -e.

\[ d \rightarrow u + W^- \]

Next, the W will decay quickly into other particles.

\[ W^- \rightarrow ? + ? \]

We know one of the mystery particles has to be negatively charged, and (looking at the
ultimate output products from the n\(\rightarrow\) p decay) it will be an electron. To conserve lepton
number, the other particle will have to be an antilepton, but it can't be charged, since
that has already been taken care of. It will have to be an anti-neutrino. Could it be a tau-
antineutrino? As it turns out, the particular flavor is also conserved, so it must be an
electron antineutrino so that we can have conservation of electron lepton number.

The vast majority of matter in our universe appears to come from just the first family of
quarks and the first family of leptons. From that, we can make neutrons and protons,
and combine them with electrons to make the building blocks of all ordinary matter.

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